The Poloidal Pump Mechanism in VEQF Theory: Unified Derivation of Mass, Charge, Magnetism, and Gravitational Drift

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Revision History

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v1.4 Corrections & Dimensional Verification (June 02, 2026)

- **G·K Numerical Consistency (Abstract, Eq. 33, §5.2, §6.2, §9.1–9.2):** Corrected erroneous value 6.48 × 10^{-59} to the consistent topological/empirical value 2.52 × 10^{-58} m^3/s (factor of ~3.89 error resolved). All α_G values updated to 8.40 × 10^{-27}.
- **K Definition (§5.2):** Fixed typo from "2N_d^2 h/c^2" to the correct "2^{N_d^2} h/c^2 = 512 h/c^2" (N_d = 3).
- **Abstract Force-Ratio Claim:** Replaced inaccurate "(l_p/d)^2/ξ^2 ~ 10^{-37}" with reference to the exact quantitative result in §8.2.
- **Δf Scaling Clarification (§3.3):** Added explicit paragraph explaining the Δf ≈ c/d approximation in the compact self-force form (Eq. 14) and its relation to the general scaling (Eq. 12).
- **Sign Tracking (§5.1):** Corrected algebraic sign handling in the EDG-to-acceleration derivation for full consistency (inward gravitational field).
- **Hierarchy Table & Check 4 (§8.3, §9.2):** Updated table with explicit note on the (Δf d/c)^2 correction factor; Check 4 rewritten for clarity.
- **Notation:** Standardized to h (Planck's constant) throughout for consistency with K = 2^{N_d^2} h/c^2 definition.
- **K vs K_eff:** Clarified distinction and usage in §6.2 with dedicated paragraph.

v1.3 Corrections & Dimensional Verification (May 31, 2026) — Retained

- **Algebraic & Dimensional Consistency (Eq. 14, 19):** Corrected the denominator in the compact self-force equation from $l_q d$ to $l_q d^3$ to ensure exact dimensional consistency (Force in Newtons) and seamless algebraic derivation of the drift velocity without forced parameters.
- **Mass-Flux Proportionality (Eq. 42):** Clarified the mass emergence proportionality using the dimensionless phase accumulation $\Phi_P^{(\text{cycle})} = \Delta f \cdot (d/c)$ to perfectly balance the mass dimension $[\text{kg}]$.
- **Electromagnetic Unit Convention (Sec. 7):** Added explicit note regarding the natural unit convention ($k_e = 1$) used for electromagnetic wake projections.
- **Sign Convention:** Maintained the v1.2 physical narrative where self-propulsion drives the soliton toward higher energy density regions (positive gradient), yielding gravitational attraction.

Abstract

We examine the poloidal pump operator (P_z) — the directed, asymmetric flux of vacuum energy quanta through VEQF solitons — as a candidate mechanism for generating multiple observed properties of matter. Within the Vacuum Energy Quanta Field (VEQF) framework, we show how the same pump flux can project onto four distinct sectors of the *external vacuum medium: (i) inertial mass via vacuum drag against beat-frequency interference, (ii) electric charge via toroidal wake circulation, (iii) magnetism via Lorentz distortion of the wake, and (iv) gravitational coalescence via gradient-asymmetric pump recoil. Two independent routes are presented for the Newtonian constant (G): one derived from pump mechanics and energy flux conservation, and another from topological invariants anchored to the Down-quark TCI. These routes are shown to be consistent with each other through the relation* $G \cdot K = 2^{10}\pi l_p^2 c \approx 2.52 \times 10^{-58}\ \mathrm{m^3/s}$ *The mesoscopic lattice scale* (l_q) *cancels in observable ratios, and the gravitational-to-electromagnetic force ratio for the proton is computed as approximately* (8.15×10^{-37})*. The Equivalence Principle emerges naturally as both inertial and gravitational responses trace to the same pump flux. This work forms part of the ongoing development of the VEQF framework and should be read in conjunction with the companion papers on hadron mass emergence and the Advanced Gravity Emergence Model.*

1. Introduction: One Engine, Four Manifestations

The Vacuum Energy Quanta Field (VEQF) theory posits a single ontological entity: a continuous, elastic, compressible medium that constitutes space itself [1–5]. Within this medium, stable elementary excitations are toroidal Hopfion solitons labeled by integer Topological Configuration Indices (TCI): $n_e = 1$ (electron), $n_U = 2$ (up quark), $n_D = 3$ (down quark), $n_S = 14$ (strange quark), and so forth [3].

The central dynamical object inside every soliton is the **poloidal pump** — a directed circulation of vacuum energy quanta driven by the helical winding of the toroidal field texture. This pump has been quantified in the Hadron Mass Emergence framework [3] through the operator P_z (Eq. 5.2) and the symmetric stiffness $\gamma = 0.08853564$ (Eq. 8.2). What has not yet been demonstrated — and what this paper establishes — is that this single pump mechanism generates all four observable manifestations of matter in the VEQF:

Manifestation	Pump Projection
Inertial Mass	Vacuum drag against inter-quark beat-frequency interference
Electric Charge	Toroidal circulation of the conical Hopfion wake
Magnetism	Moving-frame distortion of the electric wake
Gravitational Coalescence	Gradient-asymmetric pump recoil $\rightarrow$ drift up EDGs

The present derivation unifies these four phenomena by showing that each is a distinct geometric projection of the identical poloidal pump flux Φ_P onto different sectors of the external VEQF medium. No new forces, coupling constants, or mediator particles are introduced. The vacuum responds to the pump; the pump responds to the vacuum. Everything else is geometry.

2. The Poloidal Pump: Formal Definition

2.1 The Pump Operator

From the Hadron Mass Emergence framework [3, Eq. 5.2], the poloidal pump operator at position $\mathbf{r}$ in a multi-quark baryon is:

$$P_z(\mathbf{r}) = -2 \sum_{q \in B} \sigma_q N_q^2 \frac{A_q^2}{\xi_q} g\left(\frac{z - z_q}{\xi_q} \right) \tag{1}$$

where:

- q indexes constituent quarks in baryon B,
- $\sigma_q = \pm 1$ encodes poloidal spin chirality ($\sigma_U = +1, \sigma_D = -1$),
- N_q is the TCI integer,
- A_q is the strain amplitude coefficient,
- ξ_q is the characteristic Gaussian width of the quark's flux tube,
- $g(x) = \text{sech}^2(x) \tanh(x)$ is the odd shape function.

The oddness of $g(x)$ is physically decisive: $g(-x) = -g(x)$. The pump has an intrinsic direction — VEQ quanta flow from the negative-z face (intake) to the positive-z face (exhaust) of the soliton torus. This directionality is the geometric origin of all subsequent asymmetry.

2.2 Net Pump Rate per Baryon

Integrating P_z over the toroidal cross-section yields the net poloidal pump flux:

$$\Phi_P^{(B)} = \int_{\text{torus}} P_z(\mathbf{r}) \, dA \tag{2}$$

The axial pump rate Γ_B — the number of VEQ quanta processed per cycle, signed by chirality — is:

$$\Gamma_B = \sum_{q \in B} \sigma_q N_q \tag{3}$$

For the nucleons [3, Eq. 6.4]:

$$\begin{aligned}\Gamma_p &= \sigma_U N_U + \sigma_D N_D + \sigma_U N_U = (+1)(2) + (-1)(3) + (+1)(2) \\ &= 2 - 3 + 2 = \mathbf{+1} \quad \text{(proton: } UDU\text{)}\end{aligned}$$

$$\begin{aligned}\Gamma_n &= \sigma_D N_D + \sigma_D N_D + \sigma_U N_U = (-1)(3) + (-1)(3) + (+1)(2) \\ &= -3 - 3 + 2 = \mathbf{-4} \quad \text{(neutron: } DDU\text{)}\end{aligned}$$

Note: The proton pump is directed in the $+\hat{z}$ direction; the neutron pump in the $-\hat{z}$ direction with four times the magnitude. The neutron's larger pump magnitude ($|\Gamma_n| = 4$ vs $|\Gamma_p| = 1$) provides a topological basis for neutron metastability: its pump is more agitated, creating a greater disturbance in the surrounding vacuum and rendering the DDU configuration less stable against β-decay to the lower-pump UDU configuration. The sign reversal between proton and neutron also suggests a fundamental asymmetry in how these baryons interact with external EDGs — a possible origin for differentiated gravitational coupling.

2.3 Pump Flux Magnitude

The characteristic volumetric pump rate — the volume of VEQ medium processed per unit time by the poloidal circulation — is:

$$\dot{V}_P = \gamma \cdot c \cdot d^2 \tag{4}$$

where:

- $\gamma = 0.08853564$ is the symmetric stiffness (Hadron Mass v8.1, Eq. 8.2), encoding the fraction of internal strain energy channeled into poloidal circulation,
- c is the phase velocity of VEQ excitations (the propagation speed limit),
- d is the effective soliton thickness (torus minor diameter).

The energy-equivalent pump power is the product of the volumetric throughput and the background vacuum energy density ρ_{E0} (in $\mathrm{J/m^3}$, consistent with the EDG framework [4, Postulate 1]):

$$\mathcal{P}_P = \rho_{E0} \cdot \dot{V}_P = \rho_{E0}\, c\, \gamma\, d^2 \tag{5}$$

Dimensional check: $[\mathcal{P}_P] = [\rho_{E0}][c][\gamma][d^2] = (\mathrm{J/m^3})(\mathrm{m/s})(1)(\mathrm{m^2}) = \mathrm{J/s} = \mathrm{W}.$ ✓

The background energy density is $\rho_{E0} \approx 10^{-9}\ \mathrm{J/m^3}$ (equivalently $\approx 10^{-26}\ \mathrm{kg/m^3}$ as mass density) [4].

2.4 The Irrational Frustration Principle

The three irrational constants embedded in the VEQF — the golden ratio φ, the silver ratio $\delta_S = \sqrt{2} - 1$, and the fine-structure constant α — are mutually incommensurate [3, §6.6]. This means the pump can never achieve exact resonance with the vacuum. A residual phase mismatch persists perpetually:

$$\delta\phi_{\text{residual}} = \min_{p,q,r\in\mathbb{Z}} |p\ln\varphi + q\ln\delta_S + r\ln\alpha| > 0 \tag{6}$$

This Irrational Frustration guarantees that the poloidal pump is always active — the soliton can never settle into a static equilibrium with the vacuum. It continuously pumps, continuously disturbs, and continuously responds. This perpetual activity is the ontological root of both inertia (the vacuum's resistance to the pump) and self-propulsion (the pump's response to external gradients).

3. Self-Propulsion in Energy Density Gradients

3.1 The Gradient Asymmetry Mechanism

Consider a single soliton of thickness d, with its poloidal pump axis aligned along $\hat{z}$ (intake at $z = -d/2$, exhaust at $z = +d/2$). The pump drives a directed flux of VEQ quanta through the soliton body. The coupling strength between the pump and the vacuum at each point on the soliton surface is proportional to the local vacuum energy density $\rho_E(\mathbf{r})$.

Case A: Uniform vacuum ($\nabla\rho_E = 0$). The energy density is identical at all points on the soliton surface: $\rho_E(\mathbf{r}) = \rho_{E0}$. The pump couples symmetrically. Momentum exchanged with the medium at the intake and exhaust faces is equal in magnitude and opposite in direction. By Newton's third law, the soliton experiences zero net force. No drift occurs.

Case B: Energy density gradient ($\nabla\rho_E \neq 0$). The energy density at the exhaust face differs from that at the intake face. The pump couples more strongly to the denser region — more quanta are processed where the vacuum is denser. This creates an *asymmetric* momentum exchange: the face in the denser vacuum transfers more momentum to the medium, and the reaction force on the soliton is correspondingly larger on that side.

Quantitatively, let the quantum number densities (proportional to the energy density) at the two faces be $\rho_q(z_{\text{out}})$ and $\rho_q(z_{\text{in}})$, with background ρ_{q0}. To first order in the gradient:

$$\rho_q(z_{\text{out}}) - \rho_q(z_{\text{in}}) \approx d \cdot \nabla\rho_q \cdot \hat{z} \tag{7}$$

The momentum per unit time exchanged with the medium at each face scales with the local quantum density:

$$\dot{p}_{\text{in}} = \Phi_P^{(\text{quanta})} \cdot \frac{h}{l_q} \cdot \frac{\rho_q(z_{\text{in}})}{\rho_{q0}} \tag{8}$$

$$\dot{p}_{\text{out}} = \Phi_P^{(\text{quanta})} \cdot \frac{h}{l_q} \cdot \frac{\rho_q(z_{\text{out}})}{\rho_{q0}} \tag{9}$$

The net momentum transferred to the medium per unit time (in the $+\hat{z}$ direction) is:

$$\Delta\dot{p} = \dot{p}_{\text{out}} - \dot{p}_{\text{in}} = \Phi_P^{(\text{quanta})} \cdot \frac{h}{l_q} \cdot \frac{d \cdot \nabla\rho_q \cdot \hat{z}}{\rho_{q0}} \tag{10}$$

By Newton's third law, the soliton experiences an equal and opposite reaction. **The face in the denser vacuum exchanges more momentum with the medium; the reaction drives the soliton toward that face.** If the exhaust faces higher density ($\nabla\rho_q \cdot \hat{z} > 0$), then $\dot{p}_{\text{out}} > \dot{p}_{\text{in}}$, the net momentum to the medium is $+\hat{z}$, and the reaction on the soliton is also $+\hat{z}$ — *toward the exhaust, toward higher density*. If the intake faces higher density ($\nabla\rho_q \cdot \hat{z} < 0$), the net reaction is $-\hat{z}$ — again toward higher density. The force always points in the direction of $+\nabla\rho_E$:

$$\boxed{\mathbf{F}_{\text{self}} = +\Delta\dot{p}\,\hat{z} = +\Phi_P^{(\text{quanta})} \cdot \frac{h}{l_q} \cdot \frac{d}{\rho_{q0}} \cdot (\nabla\rho_q \cdot \hat{z})\,\hat{z}} \tag{11}$$

Since $\rho_q \propto \rho_E$ and $\rho_{q0} \propto \rho_{E0}$, and the quantum pump rate $\Phi_P^{(\text{quanta})} \propto \gamma\Delta f$, we obtain the scaling form:

$$\boxed{\mathbf{F}_{\text{self}} \propto +\gamma \cdot \Delta f \cdot d \cdot \frac{h}{l_q} \cdot \frac{\nabla\rho_E}{\rho_{E0}}} \tag{12}$$

The force direction is toward higher energy density — i.e., toward other masses. This is the microscopic origin of gravitational attraction.

3.2 Physical Intuition

The soliton's pump couples to the vacuum like a swimmer's limbs couple to water. In a fluid of varying density, a swimmer gets more traction in the denser region — each stroke displaces more fluid and generates a larger reaction force. The net drift is toward the denser region. Similarly, the soliton's poloidal pump processes more VEQ quanta per cycle on the side facing denser vacuum, creating an asymmetric impulse that drives the soliton up the energy density gradient. This is not a Newtonian force transmitted across space; it is geometric self-alignment driven by the pump's asymmetric coupling to a non-uniform medium.

3.3 Compact Form

Introducing the pump-vacuum coupling parameter:

$$\lambda_P \equiv \gamma \cdot d^3 \tag{13}$$

the self-force can be written compactly as:

$$\mathbf{F}_{\text{self}} = +\lambda_P \cdot \frac{\nabla \rho_E}{\rho_{E0}} \cdot \frac{hc}{l_q d^3} \tag{14}$$

Up to a dimensionless geometric factor of order unity, this is the complete self-propulsive force. The $+$ sign encodes the essential physics: the soliton accelerates toward higher vacuum energy density.

Clarification on pump-rate scaling (v1.4): The compact self-force (Eq. 14) employs the characteristic scaling $\Delta f \approx c/d$ (inverse light-crossing time of the soliton) for the effective quanta processing rate $\Phi_P^{(\text{quanta})}$. This approximation is algebraically convenient and consistent with the volumetric pump rate $\dot{V}_P = \gamma c d^2$ (Eq. 4) and the phase accumulation used in mass emergence (Eq. 42). The general scaling form in Eq. (12) retains the explicit Δf dependence for transparency. The drift velocity (Eq. 21) keeps the full Δf dependence for generality across soliton species. Higher-order corrections from exact beat-frequency locking are reserved for future VEQF dynamic simulations.

4. Vacuum Drag and Terminal Drift Velocity

4.1 The Origin of Drag

The same vacuum medium that the pump acts upon also provides resistance to motion. When a soliton translates at velocity $\mathbf{v}$, it disturbs the phase coherence that sustains its own topological stability. The vacuum must continuously re-establish the soliton's standing-wave boundary conditions against the background. This is the identical process that manifests as inertia when the soliton is externally accelerated [3, §4].

The drag coefficient is dimensionally:

$$\eta \propto \frac{m}{\tau_{\text{coh}}} \tag{15}$$

where τ_{coh} is the vacuum coherence restoration timescale. For a soliton of characteristic size d, the vacuum responds at the propagation speed c, giving $\tau_{\text{coh}} \sim d/c$ (representing the macroscopic timescale required for the vacuum to restore the soliton's global topological boundary conditions during uniform translation). The mass $m = K\Delta f$ (Hadron Mass v8.1, Eq. 4.1), so:

$$\eta \propto \frac{K\Delta f}{d/c} = K\Delta f \cdot \frac{c}{d} \tag{16}$$

The drag force is the Stokes-type law:

$$\mathbf{F}_{\text{drag}} = -\eta\,\mathbf{v} \tag{17}$$

4.2 Steady-State Drift

At steady state, the self-propulsive force balances the drag:

$$\mathbf{F}_{\text{self}} + \mathbf{F}_{\text{drag}} = 0 \tag{18}$$

From Eqs. (14) and (16):

$$+\lambda_P \cdot \frac{\nabla \rho_E}{\rho_{E0}} \cdot \frac{hc}{l_q d^3} - K\Delta f \cdot \frac{c}{d} \cdot \mathbf{v}_d = 0 \tag{19}$$

Solving for the drift velocity:

$$\boxed{\mathbf{v}_d = +\frac{\lambda_P \cdot h}{K\Delta f \cdot l_q d^2} \cdot \frac{\nabla \rho_E}{\rho_{E0}}} \tag{20}$$

Substituting $\lambda_P = \gamma d^3$:

$$\boxed{\mathbf{v}_d = +\frac{\gamma \cdot d \cdot h}{K\Delta f \cdot l_q} \cdot \frac{\nabla \rho_E}{\rho_{E0}}} \tag{21}$$

The $+$ sign is crucial: the drift velocity is parallel to the energy density gradient, meaning solitons drift toward higher ρ_E — i.e., toward other masses. For a spherical source, $\nabla \rho_E$ points radially inward (toward the source), so $\mathbf{v}_d$ is also radially inward: gravitational infall.

4.3 Comparison with the GEM Drift Law

The Advanced Gravity Emergence Model [4, Postulate 5] posits a phenomenological Fick-type drift:

$$\mathbf{v}_d = +\frac{D}{\rho_{E0}} \nabla \rho_E \tag{22}$$

Comparing Eq. (21) with (22), we identify the diffusivity in terms of pump parameters:

$$\boxed{D = \frac{\gamma \cdot d \cdot h}{K\Delta f \cdot l_q}} \tag{23}$$

This is a derived expression — not a postulated one. Every quantity on the right-hand side is fixed by the Hadron Mass framework:

- $\gamma = 0.08853564$ [3, Eq. 8.2]

- $d \approx 0.864$ fm [3, from $d/a_0 = 0.003000$]
- $K = 2^{N_d^2} h/c^2 = 512h/c^2 = 3.77472 \times 10^{-48}$ kg · s [3, Eq. 4.1]
- $\Delta f = |f_d - f_u|$ is the inter-quark beat frequency
- $l_q = 1.0 \times 10^{-20}$ m [4, Postulate 1]

5. The Gravitational Constant: Two Convergent Derivations

We now establish the expression for the Newtonian constant G within the VEQF framework. Two independent derivations converge on the same topological invariant, providing a powerful internal consistency check.

5.1 Derivation I: Pump-Mechanical (Poloidal Drift)

Consider a population of N identical solitons, each of mass m_0, distributed with number density $n(\mathbf{r})$. Each soliton's poloidal pump deposits energy into the surrounding VEQF at a characteristic rate κ (power per soliton). From Eq. (5), the pump power scales as:

$$\kappa = \zeta_\kappa \cdot \mathcal{P}_P = \zeta_\kappa \cdot \rho_{E0}\, c\, \gamma\, d^2 \tag{24}$$

where ζ_κ is a dimensionless geometric factor encoding the fraction of pump power radiated into the far-field EDG.

By energy flux conservation in three dimensions, the energy flux density at distance r from a spherically symmetric distribution is:

$$J_E(r) = \frac{\kappa N_{\text{enc}}(r)}{4\pi r^2} \tag{25}$$

The energy flux relates to the energy density gradient through Fick's law in the vacuum medium (with the sign convention that J_E is radially outward and $\nabla \rho_E$ points inward toward the source):

$$J_E = -D\, \nabla \rho_E \quad \Longrightarrow \quad \nabla \rho_E(r) = -\frac{J_E(r)}{D} = -\frac{\kappa N_{\text{enc}}(r)}{4\pi D r^2} \tag{26}$$

This is consistent with the EDG framework [4]: for a source at the origin, $\nabla \rho_E$ points radially inward (negative radial direction), and J_E is radially outward.

The gravitational acceleration experienced by a test soliton follows from the GEM drift law (Eq. 22) divided by the coherence time $\tau_{\text{coh}} = l_q/c$ [4, Postulate 5]:

$$\mathbf{g}(r) = \frac{\mathbf{v}_d}{\tau_{\text{coh}}} = \frac{+(D/\rho_{E0})\nabla\rho_E}{l_q/c} \tag{27}$$

Substituting $\nabla\rho_E$ from Eq. (26) (and noting that both $\mathbf{v}_d$ and $\nabla\rho_E$ are negative in the outward radial basis, yielding inward acceleration):

$$g(r) = \frac{D}{\rho_{E0}} \cdot \frac{\kappa N_{\text{enc}}(r)}{4\pi D r^2} \cdot \frac{c}{l_q} = \frac{\kappa\, c\, N_{\text{enc}}(r)}{4\pi\rho_{E0}\, l_q\, r^2} \tag{28}$$

The magnitude $g(r)$ is positive (inward, toward the source), consistent with gravitational attraction. Substituting $N_{\text{enc}} = M_{\text{enc}}/m_0$, $m_0 = K\Delta f$, and κ from Eq. (24):

$$g(r) = \frac{(\zeta_\kappa \rho_{E0} c \gamma d^2) \cdot c \cdot M_{\text{enc}}}{4\pi\rho_{E0}\, l_q\, r^2 \cdot K\Delta f} = \frac{\zeta_\kappa \gamma d^2 c^2}{4\pi\, l_q\, K\, \Delta f} \cdot \frac{M_{\text{enc}}}{r^2} \tag{29}$$

Comparing with Newton's law $g(r) = GM_{\text{enc}}/r^2$, we obtain the pump-mechanical expression for G:

$$\boxed{G_{\text{pump}} = \frac{\zeta_\kappa\, \gamma\, d^2\, c^2}{4\pi\, l_q\, K\, \Delta f}} \tag{30}$$

Dimensional check:
$[G] = [\zeta_\kappa \gamma d^2 c^2]/([l_q][K][\Delta f]) = (\mathrm{m}^2 \cdot \mathrm{m}^2/\mathrm{s}^2)/(\mathrm{m} \cdot \mathrm{kg} \cdot \mathrm{s} \cdot \mathrm{s}^{-1}) = \mathrm{m}^4/\mathrm{s}^2/(\mathrm{m} \cdot \mathrm{kg}) = \mathrm{m}^3/(\mathrm{kg} \cdot \mathrm{s}^2)$. ✓

5.2 Derivation II: Topological (G·K Invariant)

Independently, the VEQF framework expresses G in terms of the dimensionless coupling α_G, the lattice scale l_q, the propagation speed c, and the mass-emergence factor K [5]:

$$G = \alpha_G \cdot \frac{l_q^2 c}{K} \tag{31}$$

By equating this with the Standard Model / Planck-scale expression $G = 2\pi l_p^2 c^3/h$, and substituting the TCI-anchored K-factor $K = 2^{N_d^2} h/c^2 = 512h/c^2$ (where $N_d = 3$ is the down quark TCI), the mesoscopic scale l_q cancels entirely [5]:

$$\alpha_G = 2^{10}\pi \left(\frac{l_p}{l_q}\right)^2 \tag{32}$$

$$\boxed{G \cdot K = 2^{10}\pi\, l_p^2\, c \approx 2.52 \times 10^{-58}\ \mathrm{m}^3/\mathrm{s}} \tag{33}$$

This is the $G \cdot K$ topological invariant. It is anchored to the Down quark TCI ($N_d = 3 \Rightarrow 2^9 \Rightarrow 2^{10}$ after equating), depends only on l_p and c, and is independent of the auxiliary scale l_q.

5.3 Consistency Between Derivations

The two expressions for G — Eq. (30) and Eq. (31) with (32) — must be equal. This yields a non-trivial constraint linking pump parameters to Planck-scale geometry:

$$\frac{\zeta_\kappa \gamma d^2 c^2}{4\pi\, l_q\, K\, \Delta f} = 2^{10}\pi\left(\frac{l_p}{l_q}\right)^2 \cdot \frac{l_q^2 c}{K} \tag{34}$$

The factors of K and l_q partially cancel, leaving:

$$\boxed{\zeta_\kappa\, \gamma\, d^2\, c = 2^{12}\pi^2\, l_p^2\, l_q\, \Delta f} \tag{35}$$

This consistency condition connects the pump parameters (γ, d, Δf) of the hadron-scale soliton to the Planck scale (l_p) and the mesoscopic lattice (l_q). With $\Delta f \sim c/d$ (the characteristic beat frequency approximates the inverse light-crossing time of the soliton), Eq. (35) reduces to the scaling:

$$\zeta_\kappa \gamma d^3 \sim 2^{12}\pi^2 l_p^2 l_q \quad \Rightarrow \quad \gamma \sim \frac{2^{12}\pi^2}{\zeta_\kappa} \cdot \frac{l_p^2 l_q}{d^3} \tag{36}$$

This scaling reveals that the symmetric stiffness γ is fundamentally a ratio of Planck-scale vacuum grain volume ($l_p^2 l_q$) to soliton volume (d^3). The numerical value $\gamma \approx 0.09$ is not an independent parameter — it is geometrically determined by the hierarchy between the Planck grain and the hadron-scale soliton. The pump derivation and the topological derivation are two views of the same underlying geometry.

Key insight: The l_q cancellation in the $G \cdot K$ invariant (Eq. 33) confirms that the mesoscopic lattice scale is an auxiliary computational bridge, not a fundamental constant of nature. The product $G \cdot K$ is a topological invariant fixed by the Down quark TCI ($N_d = 3$) and Planck-scale constants alone. Any variation in G or K across different vacuum energy density environments must satisfy $G \cdot K = \text{constant}$, preserving particle identity (TCI integers) everywhere — consistent with the observed stability of quark mass ratios [5, §4].

6. The G–K Constitutive Relation: Derived, Not Postulated

6.1 Dimensional Identity

From the GEM [4, §3] and the G–K bridge document [5, §2]:

$$[G] = \frac{\mathrm{m}^3}{\mathrm{kg}\cdot\mathrm{s}^2} = \frac{\mathrm{m}^3/\mathrm{s}}{\mathrm{kg}\cdot\mathrm{s}} = \frac{[\text{Volumetric Flow Rate}]}{[K]} \tag{37}$$

This identity is dimensionally exact. The volumetric flow rate $J_q = l_q^2 c$ is the characteristic throughput of the vacuum lattice, and K is the vacuum's spectral impedance — its resistance to mass creation from frequency interference.

6.2 Physical Interpretation

With the topological invariant of Eq. (33), the dimensionless coupling is:

$$\boxed{\alpha_G = \frac{G \cdot K}{l_q^2 c} = 2^{10}\pi\left(\frac{l_p}{l_q}\right)^2 \approx 8.40 \times 10^{-27}} \tag{38}$$

- K **(kg · s):** the vacuum's stiffness against mass creation — how many kilograms of inertial mass are encapsulated per hertz of beat-frequency interference.
  $K = 2^{N_d^2} h/c^2 = 512h/c^2 = 3.77472 \times 10^{-48}$ kg · s (cosmological value).
- G **($\mathrm{m}^3/(\mathrm{kg} \cdot \mathrm{s}^2)$):** the vacuum's admittance for mass transport — how efficiently the medium conducts already-formed solitons along energy density gradients.
- $G \cdot K = 2^{10}\pi l_p^2 c = 2.52 \times 10^{-58}\ \mathrm{m}^3/\mathrm{s}$**:** the vacuum's invariant volumetric throughput. This product is a property of the medium, not of any particular soliton species. It is anchored to the Down quark TCI ($N_d = 3$) and is as fundamental to the VEQF as c and h.
- $\alpha_G = 2^{10}\pi(l_p/l_q)^2$**:** the dimensionless pump-vacuum coupling. It is *not* a free parameter — it is geometrically determined by the ratio of Planck area to lattice-cell area. With $l_p = 1.616255 \times 10^{-35}$ m and $l_q = 1.0 \times 10^{-20}$ m, the geometric expression yields exactly $\alpha_G \approx 8.40 \times 10^{-27}$, matching the value derived from measured G, K, and l_q.

Clarification on K vs K_eff (v1.4): The cosmological stiffness $K = 512h/c^2$ is the fundamental vacuum parameter used in topological relations and most derivations. An effective value $K_{\text{eff}} \approx K/3.89$ appears when fitting to the proton mass alone (arising from beat-multiplicity or silver-ratio locking effects). In this version, all topological invariants and numerical checks use the fundamental cosmological K. The small residual difference between K and K_{eff} is noted as an open detail for future refinement (see §10.2).

6.3 The Deep Meaning

K and G are not independent fundamental constants. They are the two faces of a single vacuum response:

$$\text{Vacuum} \begin{cases} \xrightarrow{\text{resists mass creation}} K \\ \xrightarrow{\text{permits mass transport}} G \end{cases}$$

This is why all attempts to quantize gravity have failed: gravity is not a force to be quantized. It is a transport process in a medium whose impedance is already fully characterized by K. The "quantum of gravity" — the graviton — does not exist because there is nothing to quantize. Mass transport along EDGs is a classical thermodynamic drift in a continuous elastic medium.

7. Unity with Electromagnetism: The Wake Coupling

7.1 Charge as Anisotropic Wake Projection

From the Electron Properties derivation [6], electric charge emerges as:

$$e = \sqrt{Kc^3} \cdot \xi(\mathcal{R}_e) \tag{39}$$

where $\xi(\mathcal{R}_e)$ is the wake coupling — the fraction of the soliton's internal dynamics that couples to the external vacuum through the conical Hopfion wake. For the TCI=1 electron:

$$\xi_e = \frac{9}{16} \cdot \frac{1}{2}(1 - \cos\theta_c) \approx 0.0014984 \tag{40}$$

with θ_c set by the 3:2 swirl-mode lock and the flux-balance cubic $\mathcal{R}_e^3/(1-\mathcal{R}_e)^2 = k_{\text{topo}}$.

The profound identity [6, §7]:

$$\frac{e^2}{r_e} = Kc^2 f_e = m_e c^2 \tag{41}$$

establishes that mass and charge are orthogonal geometric projections of the identical vacuum strain. The same topological disturbance — the TCI=1 Hopfion — produces:

- Isotropic strain $\rightarrow$ inertial mass $m_e = Kf_e$
- Anisotropic wake strain $\rightarrow$ electric charge $e = \sqrt{Kc^3}\xi_e$

> **Note on Unit Convention:** The electromagnetic relations in this framework (e.g., $e^2/r_e = m_e c^2$ and $F_{EM} = e^2/r^2$) adopt a natural unit convention where the Coulomb constant $k_e = 1$ (equivalent to $4\pi\epsilon_0 = 1$), treating charge as a direct geometric projection of vacuum strain rather than an independent SI base unit.

7.2 Magnetism as Moving Wake Distortion

When a charged soliton moves, the conical wake is compressed ahead and rarefied behind — a straightforward consequence of the finite propagation speed c of VEQ disturbances. This wake distortion is the magnetic field. Magnetism is not a separate force: it is the Lorentz-transformed electric wake [7, 8].

In the pump ontology:

$$\text{Electric field} = \text{static toroidal wake circulation from } P_z$$
$$\text{Magnetic field} = \text{moving-frame distortion of the same circulation}$$

Both are manifestations of the poloidal pump's coupling to the external vacuum, viewed in different inertial frames.

8. The Grand Unification Table

All four fundamental manifestations of matter reduce to projections of the identical poloidal pump P_z onto orthogonal sectors of the external VEQF medium:

$$\begin{array}{ll}
P_z \xrightarrow[\text{beat interference}]{\Delta f=|f_q-f_{q'}|} m = K\Delta f & \textbf{Mass} \quad (\text{isotropic vacuum drag}) \\
P_z \xrightarrow[\text{conical wake}]{\xi(\mathcal{R}_e)} e = \sqrt{Kc^3}\,\xi & \textbf{Charge} \quad (\text{toroidal wake circulation}) \\
P_z \xrightarrow[\text{Lorentz distortion}]{\mathbf{v}\neq 0} \mathbf{B} = f(\mathbf{v}, \text{wake}) & \textbf{Magnetism} \quad (\text{moving wake asymmetry}) \\
P_z \xrightarrow[\text{gradient asymmetry}]{\nabla\rho_E\neq 0} \mathbf{F}_{\text{self}} \to \mathbf{v}_d \to \mathbf{g} & \textbf{Gravity} \quad (\text{pump recoil} \to \text{drift} \to \text{coalescence})
\end{array}$$

8.1 The Equivalence Principle Emerges

Both inertial mass and gravitational coupling trace to the same pump flux:

$$\text{Inertial mass: } m = K\Delta f = \zeta_m \cdot \Phi_P^{(\text{cycle})} \cdot \frac{h}{cd} \tag{42}$$

where $\Phi_P^{(\mathrm{cycle})} = \Delta f \cdot \tau_{\mathrm{coh}} = \Delta f \frac{d}{c}$ is the **dimensionless topological phase accumulation** (beat cycles per coherence time) across the soliton, and ζ_m is a dimensionless geometric factor. Since the confinement mass term $\frac{h}{cd}$ carries units of mass $[\mathrm{kg}]$, this proportionality naturally recovers $m \propto \Delta f$, identifying the vacuum stiffness constant K as fundamentally scaling with $\frac{h}{c^2}$.

$$\text{Gravitational coupling: } \kappa \propto \gamma \rho_{E0} c d^2 \propto \Phi_P^{(\mathrm{quanta})} \cdot \frac{hc}{l_q} \tag{43}$$

Both scale linearly with the pump flux parameters. Consequently, the ratio $m/(\text{gravitational source strength})$ is a constant of the vacuum medium — not a property of individual solitons. The Weak Equivalence Principle is the statement that all solitons, regardless of TCI, couple to the vacuum through the same pump-vacuum stiffness γ.

8.2 Why Is Gravity So Weak? — The l_q Cancellation Discovery

This is perhaps the most important quantitative result of the unified pump framework. Using the topological expression for G from §5.2, the gravitational-to-electromagnetic force ratio for two identical solitons can be computed without any reference to l_q:

$$\frac{F_G}{F_{EM}} = \frac{Gm^2/r^2}{e^2/r^2} = \frac{Gm^2}{e^2} \tag{44}$$

Substitute $G = \alpha_G l_q^2 c/K$, $m = K\Delta f$, and $e^2 = Kc^3\xi^2$:

$$\frac{F_G}{F_{EM}} = \frac{(\alpha_G\, l_q^2 c/K) \cdot K^2 \Delta f^2}{Kc^3\xi^2} = \frac{\alpha_G\, l_q^2\, \Delta f^2}{c^2\, \xi^2} \tag{45}$$

Now substitute the topological $\alpha_G = 2^{10}\pi(l_p/l_q)^2$ from Eq. (32):

$$\boxed{\frac{F_G}{F_{EM}} = 2^{10}\pi \cdot \frac{l_p^2\, \Delta f^2}{c^2\, \xi^2}} \tag{46}$$

Discovery: l_q cancels completely. The gravitational-to-electromagnetic force ratio depends only on the Planck length l_p, the soliton beat frequency Δf, the propagation speed c, and the EM wake coupling ξ. The mesoscopic lattice scale — which encodes our ignorance of the vacuum's granularity — drops out of all observable ratios. This is the hallmark of a well-defined effective theory: the auxiliary scale used in intermediate steps disappears from physical predictions.

Using $\Delta f = m/K$ for a soliton of mass m, Eq. (46) becomes:

$$\frac{F_G}{F_{EM}} = 2^{10}\pi \cdot \left(\frac{l_p\, m}{c\, K\, \xi}\right)^2 \tag{47}$$

For the proton: $m_p = 1.673 \times 10^{-27}$ kg, $K = 3.77472 \times 10^{-48}$ kg $\cdot$ s, $\xi \approx 1.50 \times 10^{-3}$ the calculation yields $\approx 8.15 \times 10^{-37}$, in excellent agreement with the observed proton-proton gravitational-to-electric force ratio of 8.09×10^{-37} (0.7% difference, well within parameter uncertainty).

8.3 The Physical Origin of Gravity's Weakness

Equation (46) can be rewritten in a physically transparent form using the characteristic scaling $\Delta f \sim c/d$ (inverse light-crossing time of the soliton):

$$\boxed{\frac{F_G}{F_{EM}} \sim 2^{10}\pi \cdot \left(\frac{l_p}{d}\right)^2 \cdot \frac{1}{\xi^2} \cdot \left(\frac{\Delta f d}{c}\right)^2} \tag{48}$$

The three-factor hierarchy (with the additional explicit $\Delta f d/c$ correction) is:

Factor	Value (proton)	Physical Origin
$2^{10}\pi$	$\sim 3.2 \times 10^3$	Topological: Down quark TCI $N_d = 3 \Rightarrow 2^{N_d^2+1}\pi$
$(l_p/d)^2$	$\sim 3.5 \times 10^{-40}$	Geometric dilution: the soliton cross-section d^2 is 40 orders of magnitude larger than the Planck area l_p^2.
$1/\xi^2$	$\sim 4.4 \times 10^5$	Wake focusing: the EM wake couples through a narrow cone ($\xi \sim 10^{-3}$).
$(\Delta f d/c)^2$	$\sim 1.64 \times 10^{-6}$	Beat-frequency scaling relative to soliton light-crossing frequency.

Gravity is weak because solitons are large compared to the Planck grain, and the gravitational (monopole) projection lacks the geometric focusing of the electromagnetic (dipolar) wake. The ratio $(l_p/d)^2 \sim 10^{-40}$ is the single largest factor. This answers — quantitatively, from first principles — the longstanding puzzle of why gravity is $\sim 10^{36}$–10^{40} times weaker than electromagnetism.

9. Numerical Cross-Check

9.1 Parameters

Parameter	Symbol	Value	Source

Vacuum elasticity	K	$3.77472 \times 10^{-48}\ \mathrm{kg \cdot s}$	[3, Eq. 4.1]
Symmetric stiffness	γ	0.08853564	[3, Eq. 8.2]
Soliton thickness	d	$\sim 0.864 \times 10^{-15}\ \mathrm{m}$	[3, Eq. 8.3]
Lattice spacing	l_q	$1.0 \times 10^{-20}\ \mathrm{m}$	[4, Postulate 1]
Planck length	l_p	$1.616255 \times 10^{-35}\ \mathrm{m}$	CODATA
Newton's constant	G	$6.67430 \times 10^{-11}\ \mathrm{m^3/(kg \cdot s^2)}$	CODATA
Throughput invariant	$G \cdot K$	$2.52 \times 10^{-58}\ \mathrm{m^3/s}$	[5, §7] (verified)
EM wake coupling	ξ	1.4984×10^{-3}	[6, Eq. 41]
Background energy density	ρ_{E0}	$\approx 10^{-9}\ \mathrm{J/m^3}$	[4, Postulate 1]

9.2 Consistency Checks

Check 1 — $G \cdot K$ invariant:

$$G \cdot K = 6.67430 \times 10^{-11} \times 3.77472 \times 10^{-48} = 2.519 \times 10^{-58}\ \mathrm{m^3/s}$$

The topological prediction is $2^{10}\pi l_p^2 c = 3217 \cdot (2.612 \times 10^{-70}) \cdot 2.998 \times 10^{8} = 2.52 \times 10^{-58}\ \mathrm{m^3/s}$. Exact match.

Check 2 — α_G from l_p/l_q:

$$\alpha_G = 2^{10}\pi\left(\frac{l_p}{l_q}\right)^2 = 3217 \cdot \left(\frac{1.616 \times 10^{-35}}{1.0 \times 10^{-20}}\right)^2 = 8.40 \times 10^{-27}$$

Fully reconciled with the empirical value derived from measured constants.

Check 3 — F_G/F_{EM} ratio:

$$\left.\frac{F_G}{F_{EM}}\right|_{\text{proton}} = 8.15 \times 10^{-37} \quad \text{vs. observed } 8.09 \times 10^{-37}$$

Agreement to $\sim 0.7\%$.

10. Open Frontiers and Falsification Criteria

10.1 What Has Been Achieved

Step	Status
$P_z \to$ pump flux magnitude and direction	Quantified [3]
Baryon pump rates: $\Gamma_p = +1, \Gamma_n = -4$	Fixed (v1.1)
Gradient asymmetry $\to \mathbf{F}_{\text{self}}$ direction (toward higher ρ_E)	Derived (§3)
Self-force scaling $\propto +\gamma dh \nabla \rho_E / (l_q \rho_{E0})$	Derived (§3)
Drag coefficient $\eta \propto K \Delta f c / d$	Derived (§4)
Drift velocity $\to$ Fick-type law (toward higher ρ_E)	Derived (§4)
EDG profile $\to 1/r^2$ from flux conservation	Recovered [4]
G–K constitutive relation: $G \cdot K = 2^{10} \pi l_p^2 c$	Topological invariant [5]
$\alpha_G = 2^{10} \pi (l_p / l_q)^2$	Derived (§5.2)
Pump-mechanical G consistent with topological G	Consistency condition Eq. (35)
Mass-charge unity from identical pump	Established [6]
Equivalence Principle as emergence	Demonstrated (§8.1)
l_q cancellation $\to$ quantitative gravity weakness	Discovered (§8.2)
F_G / F_{EM} for proton: 8.15×10^{-37} (0.7% of observed)	Computed (§8.2)

10.2 What Remains Open

Frontier	Required
Exact ξ to 6+ significant figures	Higher-precision flux-balance solution for $\mathcal{R}_e$

Baryon-specific $\Gamma_B \rightarrow$ differentiated gravitational coupling	Extension of pump formalism to multi-quark EDG source terms; test whether $\lvert\Gamma_n\rvert/\lvert\Gamma_p\rvert = 4$ predicts measurable differences in neutron vs. proton gravitational response
Reconciliation of $K = 512h/c^2$ with $K_{\text{eff}} = m_p/(4\Delta f)$	Resolution of factor ~ 3.89 — possibly a beat-multiplicity correction from the silver-ratio lock
Dark matter TCI solitons as primary EDG scaffold sources	Extension to TCI = 4–13, $n > 14$
Chromatic lensing from frequency-dependent pump coupling	$\xi(\omega)$ generalization; testable prediction of frequency-dependent gravitational deflection
Precise l_q from α_G and l_p	Invert Eq. (38): $l_q = l_p\sqrt{2^{10}\pi/\alpha_G} \rightarrow$ yields l_q directly from measured constants

10.3 Falsification Criteria

The unified pump theory requires all of the following to hold:

- $G \cdot K$ must be invariant across all environments — any measured variation in either G or K individually must be compensated by the other such that $G \cdot K = 2^{10}\pi l_p^2 c$ remains constant. This is the single most stringent test of the topological anchoring.
- Charge-mass unity ($e^2/r_e = m_e c^2$) must hold as an identity, not an accident. Any deviation indicates a failure of the wake-coupling ontology.
- $\alpha = 1024\pi\xi^2$ must match CODATA α to within higher-order correction tolerance (currently 0.0006%).
- Gravitational acceleration must vanish in a perfectly homogeneous universe ($\nabla\rho_E = 0 \Rightarrow \mathbf{g} = 0$). This distinguishes the pump-drift mechanism from all force-based theories of gravity.
- Chromatic gravitational lensing must show frequency-dependent deflection angles, as the pump's EDG response depends on soliton beat frequency Δf.
- Neutron gravitational response may differ measurably from proton response due to $\lvert\Gamma_n\rvert/\lvert\Gamma_p\rvert = 4$ — a testable prediction of the corrected pump rates.

11. Conclusion

We have demonstrated that the poloidal pump operator P_z — the directed, asymmetric circulation of vacuum energy quanta through every VEQF soliton — is the single irreducible engine underlying all four observable manifestations of matter. Inertial mass, electric charge, magnetism, and gravitational coalescence are not independent phenomena requiring separate forces or coupling constants. They are orthogonal geometric projections of identical vacuum strain onto different sectors of the external VEQF medium, all driven by the same pump.

The Newtonian gravitational constant G is not a fundamental constant of nature. It is a transport coefficient — a vacuum admittance — expressing how efficiently the elastic medium conducts mass along energy density gradients given its intrinsic stiffness K against mass creation. The constitutive relation $G \cdot K = 2^{10}\pi l_p^2 c$ is a topological invariant anchored to the Down quark TCI ($N_d = 3$), unifying microphysics and gravitation within a single parameter-free ontological framework.

The mesoscopic lattice scale l_q cancels from all observable ratios — confirming its status as an auxiliary computational bridge rather than a fundamental constant. This cancellation enables the first quantitative, parameter-free computation of the gravitational-to-electromagnetic force ratio for the proton, yielding $F_G/F_{EM} = 8.15 \times 10^{-37}$, in agreement with the observed 8.09×10^{-37} to within 0.7%.

Gravity is weak — 10^{36}–10^{40} times weaker than electromagnetism — because solitons are large compared to the Planck grain: $(l_p/d)^2 \sim 10^{-40}$. The gravitational (monopole) projection of the pump samples this geometric dilution across the entire soliton cross-section, while the electromagnetic (dipolar) wake concentrates the same pump flux through a narrow cone ($\xi \sim 10^{-3}$), gaining a relative factor of $\sim 10^5$. Gravity is not a separate force fighting against electromagnetism; it is the isotropic shadow of the same engine, diluted by geometry.

The Equivalence Principle is not a postulate — it is the statement that the same pump flux determines both how much the vacuum resists a soliton's acceleration (inertia) and how strongly that soliton's pump couples to external gradients (gravitational source strength). Both trace to the identical $\gamma\Delta f$, and their ratio is a constant of the vacuum medium itself.

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